Comment on "Scaling Hypothesis for the Spectral Densities in the $O(3)$ Nonlinear Sigma Model"

Adrian Patrascioiu
Physics Department, University of Arizona,
Tucson, AZ 85721, U.S.A.

and
Erhard Seiler
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 Munich, Germany

Abstract

We comment on the recent paper by Balog and Niedermaier [1].

Balog and Niedermaier's scaling hypothesis for the spectral densities in the $O(3)$ nonlinear σ model [1] does not seem to be supported by the Monte Carlo data. As we showed in a recent paper [2], the numerics indicate that the continuum limit of the lattice $O(3)$ σ model agrees as well with the S-matrix prediction [3] as with the continuum limit of the dodecahedron spin model. In the latter model the massive high temperature phase must terminate at some finite inverse temperature β. It is well known that asymptotic freedom requires the current 2-point function to diverge for $p/m \to \infty$ [4]. However in another recent paper we proved that the current 2-point function is bounded at finite β, due to reflection positivity and a Ward identity.

Consequently, unless the observed agreement between the $O(3)$ and dodecahedron spin model is accidental and disappears at larger values of p/m,

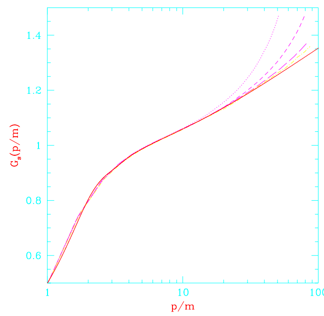

Figure 1: Monte-Carlo data and the Balog-Niedermaier prediction (solid line) for the spin two point function.

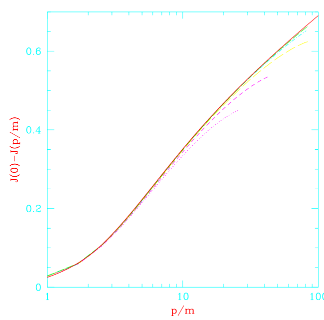

Figure 2: Monte-Carlo data and the Balog-Niedermaier prediction (solid line) for the current two-point function.

the Balog and Niedermaier scaling hypothesis, predicting a logarithmic increase of the current 2-point function with p/m, cannot be correct. Moreover, since the spin 2-point function should behave as $(p/m)^{\eta-2}$ for large p/m, the behavior of the odd and even-particle number spectral densities $\rho^{(n)}(\mu)$ must be quite different.

This different behavior is shown both by our Monte Carlo data and by the Balog-Niedermaier prediction itself, if one looks at it on a logarithmic scale (see Fig.1, Fig.2): while for the odd case (spin) the data start growing faster than logarithmically, they grow more slowly than $\ln(p/m)$ for the even case (current). This behavior gives support to our scenario of a power-like increase in the odd and boundedness in the even sector. Incidentally for the odd case the logarithmic slope at $p/m = 100$ is already .143 and growing; this is larger than the prediction $4/3\pi^2$ in [3] (the long version of their letter)

for the asymptotic slope.

Finally, Fig.1 in [3] is not representing our Monte Carlo data for $p^2G(p)$ (instead of p^2 a lattice variant is used); $p^2G(p)$ is shown in Fig.1 here (see also [2]). Also the introduction of [3] contains a misleading statement regarding our work: they give the incorrect impression that superinstantons are creating a mass gap. After pointing out, correctly, that superinstantons restore the $O(3)$ symmetry [5], they go on to state the triviality that in the absence of a KT transition, the theory has a mass gap. The way the sentences are juxtaposed, a false logical connection is suggested. In fact, via our percolation arguments [6, 7], the super-instanton gas is naturally associated with masslessness and we believe that the massive, high temperature phase, which appears to be correctly described by the Zamolodchikovs' S-matrix has nothing to do with the large β regime of the model, which is dominated by super-instantons and is massless.

References

[1] J. Balog and M.Niedermaier, hep-th/9701156, *Phys.Rev.Lett.* **78** (1997) 4151.

[2] A.Patrascioiu and E. Seiler, hep-lat/9706011

[3] J. Balog and M.Niedermaier, hep-th/9612039.

[4] J.Balog, *Phys.Lett* **B 300** (1993) 145.

[5] A.Patrascioiu and E.Seiler, *Phys.Rev.Lett* **74** (1995) 1920.

[6] A.Patrascioiu, University of Arizona preprint AZPH-TH/91-49.

[7] A.Patrascioiu and E.Seiler, *Nucl.Phys.B.(Proc.Suppl.)* **30** (1993) 184.